Is Monetary Policy Responsive to External Reserves? Empirical Evidence from Nigeria

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Abstract

The global economy has witnessed extraordinary boost in the accumulation of external reserves, following the Asian financial crisis of the 1990s. External reserves increased sharply from US$1.2 trillion in 1995 to over US$10.0 trillion in January 2012. Developing countries increased their share from 30.0 per cent in 1990 to 67.0 per cent in 2011. Nigeria is not left out in this trend, as external reserves grew from US$5.5 billion in 1999 to US$34.68 billion in March 2012, representing over 530 per cent increase within the period. This placed Nigeria as the 44th largest reserves holder in the world. Reflecting on this phenomenal increase in Nigeria's reserves, that places Nigeria in such a strategic position, there is the need to examine, if the Central Bank of Nigeria considers the changes in the level of reserves, in its monetary policy decision making. The study applied an Autoregressive Distributed Lag (ARDL) approach to an extended version of the Taylor-type rule to estimate the monetary policy reaction function for Nigeria, with emphasis on external reserves. The results show that the Central Bank of Nigeria reacts to changes in the level of external reserves and exchange rate, in addition to output gap, thereby rendering the cogent conventional Taylor rule inadequate to assess the monetary policy reaction function of the Central Bank of Nigeria. This justifies the modification of the rule to incorporate other variables in addition to inflation and output to capture the reaction of monetary policy to developments in the economy. The study also validates the interest rate smoothing behavior, showing that the Central Bank of Nigeria is concerned with costs associated with interest rate variability.

I.   Introduction

The seminal work of Taylor (1993) provides the basis for the study of monetary policy reaction function. The rule states that central bankers increase/decrease the nominal interest rate, if inflation is above/below the target (inflation gap) and/or output is above/below the potential (output gap). However, several studies[1] later proved that policymakers consider and analyze other variables before policy decisions are taken and pronounced. Therefore, the current reality is that policymakers react to as many variables as

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possible besides inflation and output gaps. This prompted the extension of the Taylor rule to incorporate other variables including macro-fundamentals (Senbet, 2011). Considering the growing influence of external reserves on central bank's policy decision making, other scholars, such as Prakash and Willi (2011) further extended the Taylor rule to include external reserves.

The global economy has witnessed a phenomenal increase in the accumulation of external reserves, following the Asian financial crisis of 1997/98. Most of these reserves are from the developing countries. Central banks' holding of external reserves has escalated sharply from US$1.2 trillion in 1995 to over US$10.0 trillion in January 2012. Developing countries have increased their share of the reserves from 30.0 per cent in 1990 to 67.0 per cent in 2011. In Africa, external reserves increased from US$39.0 billion in 1995 to over US$600.0 billion in 2011. Nigeria also follows suit, as external reserves grew from a mere US$5.5 billion in 1999 to US$34.68 billion in March 2012, representing over 530.0 per cent increase within the period. Thus, as at March 2012 Nigeria was the 44th largest reserves holder in the world (AllAfrica.com, March 14, 2012).

Reflecting on this phenomenal increase in Nigeria's reserves, that places Nigeria in such a strategic position, there is the need to examine, if the monetary authority considers this colossal changes in the level of reserves, in its monetary policy decision making. In other words, does the Central Bank of Nigeria respond or react to changes in the level of external reserves? This study is, therefore, an attempt to examine the link between external reserves and monetary policy with emphasis on the reactionary tendencies of monetary policy, to not only changes in the general price level and output, but also to changes in external reserves accumulation. This will, to a large extent, assist policy makers and players in the financial market, predicts the possible future path of monetary policy, using developments in both the domestic economy and in the economy of Nigeria's major trading partner.

To achieve this, the paper is structured into five sections. After this introduction, section two reviews relevant literatures and provides the basis for the extension of the Taylor-type rule monetary policy reaction function. Section three, describes the data source and methodology, while section four presents the results and section five concludes the paper.

II.   Relevant Literature

II.1   Empirical Literature

There exists a large body of literature on external reserves that attempted to find the determinants of reserves holdings, as well as its optimal level. From the earlier

work of Heller (1966) to that of Aizenman and Marion (2003) and David and Yaaba (2011). There is also a great chunk of literature on the estimation of monetary policy rule in line with the work of Taylor (1993). This reflects the notion of at least, short run impact of monetary policy on the economy as against the monetary neutrality view of some monetarists. The simplest version of the Taylor rule (1993) entails that the short term interest rate should positively respond to the rate of inflation and output gap. In this regards, therefore, scholars attempted to estimate monetary policy reaction in this direction.

Douglas (2010) estimates monetary policy reaction function for OECD countries and concluded that it can provide insight into the factors influencing monetary policy decisions, but observes that differences exist across countries as to whether monetary policy reacts solely to expected inflation or takes into account expected output developments. A range of other factors, such as monetary policy in other large economies, can also influence monetary policy reactions in smaller countries. He further confirmed that monetary policy reacts less to contemporaneous measures of the output gap, while developments in asset prices do not have any influence on monetary policy decisions.

Doladoy et al (2003), investigates the implication of a non-linear Phillips curve for the derivation of optimal monetary policy rules for four European countries and the US. They opined that, the optimal policy is nonlinear, with the policymaker increasing interest rates by a larger amount when inflation/output is above target compared to the reaction when inflation/output is below target. Specifically, the model predicted that such a source of nonlinearity leads to the inclusion of the interaction between expected inflation and the output gap in an otherwise linear Taylor rule.

James et al (2007), modeled the US economy using vector autoregressive (VAR) model. The data spanned from 1984 to 2003. They found that, contrary to the official claims; the Federal Reserve neither targets inflation nor reacts to its signals, but rather unemployment signals in a way that suggests that fear of unemployment is a driving force of monetary policy decisions. They tested the variations in the workings of the Taylor rule by using data dating back to 1969 to run what they referred to as "dummy variable regressions". The results suggest that after 1983 the Federal Reserve stop reacting to inflation or high unemployment, but low unemployment. They further show that monetary policy, measured by yield curve, has significant causal effect on pay inequality. The results also provide evidence of partisan bias in monetary policy decisions, particularly during election years.

Kontonikas and Montagnoli (2004) examined the relationship between monetary policy and assets prices in the UK in the context of empirical policy rules, using data from 1992-2003. They find that UK policymakers take into account the effect of assets price inflation in setting interest rate with a higher weight to property market fluctuations. Asset-inflation augmented rules describe relatively more accurately the actual policy and the results are robust to modeling the effect of the Bank of England independence.

Jesus and Manuel (2008) adopted a simple ordered probit model to analyze the monetary policy reaction function of the Colombian Central Bank (CCB). The result provides evidence of an asymmetric reaction in the sense that, the central bank increases the bank rate when the gap between observed inflation and the inflation target is positive, but did not reduce the bank rate when the gap is negative, suggesting that the bank is more interested in achieving the announced inflation target than in reducing reasonable inflation rate. The forecasting performance of the model, both within and beyond the estimation period was sound.

Bernd and Boris (2005) estimated the monetary policy reaction functions for the Bundesbank and European Central Bank (ECB), using monthly data that spanned from 1979:4 – 1998:12; and 1999:1 – 2004:5, respectively. The results show that, while the ECB and the Bundesbank react almost the same way to inflation expectations, the ECB reacts relatively stronger than Bundesbank to output gap. According to the authors, theoretical considerations suggest that the stronger response to the output gap was due to higher interest rate sensitivity of the German output gap than to a higher weight attached to the output stabilization by the ECB. Counterfactual simulations carried-out based on the estimated interest rate reaction function proves that German interest rates would not have been lower under a hypothetical Bundesbank regime after 1999. However, their conclusion was said to be crucially dependent on the assumption of an unchanged long-run real interest rate for the crisis period.

Koiti and Naoyuki (2007) empirically analysed the Japanese monetary policy based on time-varying structural vector auto-regressions (TVSVAR). The TVSVAR is a dynamic full recursive structural VAR that includes a monetary policy reaction function, an aggregate supply function, an aggregate demand function and an effective exchange rate determination function. They used quarterly data on nominal short term interest rate, inflation rate, real growth rate and nominal effective exchange rate and the results show that monetary policy in Japan was ineffective in the 1990s.

Adanur-Aklan and Nargelecekenler (2008) estimated both backward-looking and forward-looking monetary policy reaction function of the Central Bank of the Republic of Turkey (CBRT) with focus on the post-crisis period between August 2001 and September 2006. The study laid emphasis on inflation targeting and the results show that CBRT complied with the Taylor rule in setting interest rate. The coefficient of inflation and output gap was greater in the forward-looking model than in the backward-looking model. Although, the results of the forward-looking model mirrors the monetary policy decisions in Turkey, but the expected inflation which appeared to be the main variable that reacts enormously to the policies of CBRT, suggests to some extent, the non-accommodating nature of monetary policy at least over the post-crisis period. They concluded that, in general, monetary policy based on Taylor rule was effective in inflation targeting in Turkey.

Yazgan and Yilmazkuday (2007) estimated only a forward looking monetary policy rule for Israel and Turkey with inflation target as priority against fixed target adopted in earlier research for developed countries. They asserted that a forward-looking Taylor rule provides reasonable description of central bank behavior in both countries. They concluded that monetary policy was more effective in both countries, especially in Turkey, when compared to the developed countries.

Sanchez-Fung (2002) estimated a hybrid monetary policy base reaction function for the Dominican Republic (DR). The estimated reaction function shows that the central bank was biased towards targeting the gap between the parallel market and official exchange rate. This bias was more systematic after the mid-1980s. According to him, the findings of the study confirm the long standing endorsement of a multiple exchange rate regime by the Central Bank of the Dominican Republic, supporting the learning process hypothesis for the bank.

Inoue and Hamori (2009) applied a dynamic ordinary least square (DOLS) model in estimating India's monetary policy reaction function. He constructed an open-economy version of Taylor rule using monthly data from the period April 1998 to December 2007. While the sign of the coefficient of output gap was found to be consistent with theory and statistically significant, that of inflation was not. The estimation of Taylor rule with exchange rate yielded a statistically significant and theoretically consistent coefficient of output gap and exchange rate. They, therefore, concluded that, inflation rate did not play a significant role in the conduct of monetary policy in India, during the studied period. Thus, they recommended that India should adopt an inflation-target type policy framework.

Malik and Ahmed (2008) estimated a slightly modified version of the Taylor-type reaction function for Pakistan. The results indicate a pro-cyclical response of State

Bank of Pakistan (SBP) to economic fluctuations. They proved that the emphasis of the SBP was more on economic growth than price stability and output stabilization. The study also exposed the policy inconsistency, as some parameters in the reaction function show variation in recursive estimation.

Siri (2009) analyzed the reaction function of the Central Bank of Ghana and Nigeria and the West African Economic and Monetary Union (WAEMU). The empirical result suggests that Ghana and Nigeria's monetary policy are not consistent with the monetary policy rule according to the original Taylor model or to most of the adjusted variants of the model. Interest rate reacts weakly to the variations in inflation and output gap. In the case of WAEMU, the central banks seem to apply a Taylor rule adjusted by the interest rate of France.

Iklaga (2008) used a simple Taylor-type model to estimate a reaction function for the Central Bank of Nigeria (CBN). The results show that a Taylor-rule framework summarizes well the key elements of monetary policy in Nigeria. Inflationary pressures and output were proved to be the driving force of monetary policy decisions of the CBN.

Agu (2007) used two models to estimate the monetary policy reaction function for the Central Bank of Nigeria (one based on the historical process of the Central Bank of Nigeria, while the other adheres to the Taylor-type rule). The results validate the importance of inflation in the monetary policy reaction in Nigeria. It also shows that ex-ante pronounced policy target of the central Bank of Nigeria (CBN) differs from ex-post outcomes. The result, however, did not confirm the interest rate smoothing behavior, showing that the CBN is not critically concerned with costs associated with interest rate variability or that the costs have been swamped by other policy challenges. The results did not also confirm either the thorny issue of fiscal dominance or indications of significant impact of oil prices on the monetary policy framework.

However, despite all these extensions, only a few of the studies on the estimation of monetary policy rule considered the influence of external factors in the monetary policy reaction function. Good examples of such studies are; Berument and Tasci (2004) which used the difference between interbank money market rate and the depreciation rate and found that monetary policy significantly reacts to changes in foreign reserves in Turkey.

In the same vein, Prakash and Willi (2011) examined monetary policy response to international reserves for some selected East Asian countries by extending the Taylor rule. They employed an autoregressive distributed lag (ARDL) approach and the result indicates that the countries, in addition to macroeconomic stability, consider external constraint and financial stability in their monetary

policy decisions. Thus, monetary policy significantly reacts to international reserves, particularly after the Asian financial crisis of the 1990s. They, therefore, concluded that the conventional Taylor-type rule did not properly capture the monetary policy reaction in the emerging economies of Asia.

Pei-Tha and Kian-Teng (2010) extended the structural vector auto-regression (SVAR) model based on Svensson (2000) to confirm the notion that monetary policy is not only concerned with output gap and inflation, but also external influences. Using data from Indonesia, Malaysia and Thailand, the results indicate that the sampled countries do not consider exchange rate and terms of trade in their monetary policy objectives, although they affect the actions of the central banks. They also confirmed the usefulness of interest rate as a measure of the counter-cyclical policies of the central banks.

II.2   Monetary Policy Rules: Taylor's Framework

Two distinct types of monetary policy instrument rules exist in literature; money growth and interest rate setting. The first was predominant in the 1980s and mostly referred to as Friedman's framework. The latter is associated with Taylor (1993). The Friedman's framework was also influenced by the Quantity Theory of Money (QTM). The New Keynesians concern of the unstable velocity of money, as well as the worry of the Post Keynesians about the endogeneity of money supply, led to the gradual abandonment of Friedman's framework by many advanced and emerging economies. The Taylor framework based on the seminal work of Taylor (1993) proposes a rule of setting the interest rate with a specific parameterization as a sum of the equilibrium real rate (r*), inflation gap and output gap (y_t), such that:

$$i = r^* + \pi_t + 0.5\left(\pi_t - \pi^*\right) + 0.5 y_t$$

(1)

Where i is nominal federal funds rate, r* is the equilibrium real interest rate, π_t is the inflation rate over the previous four quarters, π^* is the inflation target and y_t is the per centage of deviation of real gross domestic product (GDP) from its target. With a predetermined real interest rate and the inflation target equaling 2, the rule of setting interest rate befits[2]: $i_t = 1 + 1.5\pi_t + 0.5 y_t$.

Taylor (1993) deduced a rule-based framework based on the observed behavior of the Fed as it affects inflation and growth of risks. He argues that the rule can be derived from the quantity theory of money, assuming a constant velocity of

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[2]*Based on calibration, Taylor found that a rule with the parameters set to r* = π* = 2 and f_π = ϕ_y = 0.5 traces fairly well, the Fed funds rate between 1987 and 1992.*

money. Semmler et al. (2005), opined that the Taylor rule can also be obtained by solving a dynamic optimization problem through minimizing the central bank's loss function that includes both inflation and output gaps. Standardizing it further, they proved that in a dynamic decision problem, the arbitrary weights proposed by Taylor (i.e. 1, 1.5 and 0.5) can be obtained from the coefficients of the IS and Phillips curves. This position corroborated the earlier work of Svensson (2000). The rule attracted the attention of some policy makers, because, it was able to fairly predict the federal funds rate in the late 1990s.

This simple original framework of the Taylor rule is marred by many shortcomings. Prominent among them are: first, considering the changing role of central banks, variables other than the inflation and output gaps are as important for monetary policy decision making. These variables include real exchange rate, terms of trade, outputs of trading partners, interest rate of other countries and asset price. Second, other critical information about the economy are not captured in the rule as it dwells mainly on a closed economy. Third, Ball (1999) argues that the optimal monetary policy rule in an open economy is a function of not only short-term interest rate, but also exchange rate. This is because of the popular exchange rate channel of monetary policy transmission. Fourth, the rule is 'developed country bias' and what is suitable for developed countries may not necessarily fit the emerging and developing countries. Fifth, the Asian financial crisis of the 1990s, as well as the mortgage crisis that started from the US in 2007 proved the failure of the rule to avert crisis.

Other scholars such as Cecchetti (2000) argued that even the interest rate smoothing concept of the monetary policy reaction function which is presumed to be enough for keeping the chances of financial crisis to a minimum failed to curb financial crisis. In a nutshell, even the inflation targeting rule in the conventional Taylor-type rule, ignores the unavoidable external constraint faced by countries which are not issuing reserve currency. Following these weaknesses, therefore, an extension to Taylor rule became crucial.

II.3   An Extended Monetary Policy Reaction Function

Considering the inherent weaknesses of the analyzed Taylor-type rule and following the recent work of Prakash and Willi (2011), this study adopts a simple variant of the reaction function to examine the response of monetary policy in Nigeria to the level of external reserves, besides inflation and output changes. The extension of the monetary policy reaction function for Nigeria, therefore, can take the form below:

$$IR_t = \beta_0 + \beta_1 \log R_t + \beta_2 IF_t + \beta_3 Y_t + \mu_t \tag{2}$$

Where IR_t is the short-term interest rate, R_t represents the external reserves, IF_t is inflation, Y_t is the output gap. The *apriori* expectations of the variables, from theoretical stand point are that all the coefficients, except output gap, will take positive signs. Another expectation from the theoretical stand-point is that, the long-run coefficient of inflation should preferably exceed one. This guarantees that the Taylor rule equates inflation to its target. However, since the consideration here is not inflation gap (as advocated in the Taylor rule, the coefficient shall be implicitly determined in the estimation and not assign *apriori*. The magnitude of the coefficient on the output gap Y_t, following Prakash and Willi (2011), depends on the slope of the aggregate supply curve and the variability of output in the loss function.

In this specification, inflation is not the main focus as in the original model. The exchange rate covertly assumes the impact of the changes in exchange rate on inflation rate. The external reserves is assumed to be necessary to minimize the likely output costs in case of break out of financial crisis, when capital flows is volatile. It will also be used to maintain stability in exchange rate, so as to enhance economic growth, as well as creates employment. Short-term interest rate is retained to proxy monetary policy instrument, since both the Post Keynesians and the New Keynesians still consider short-term interest rate as exogenous, but somehow set by the central banks (Lavoie, 2006; Cecchetti, 2000). Central banks, particularly in emerging and developing countries do not rely solely on policy rate, as they occasionally intervene in the money markets to influence the short-term interest rate.

In estimating the monetary policy reaction function, the log level of external reserves is considered as against the deviation of the level of external reserves from any target used in some studies. This is, because, there is still no consensus as to what constitutes an optimum level of external reserves, hence, optimum reserves is country specific depending on the macroeconomic and political objectives of the country. Similarly, some internationally accepted ratios, such as the IMF rule of thumb and the WAMZ convergence criteria of 3 months of imports cover, the Greenspan-Guidotti's rule of covering short-term external debt, as well as the Shcherbakov approach of 5-20 per cent of broad money supply (M_2) are also not highly relevant here, because, of their myopic emphasis. They lay emphasis on either, the current account or capital account. Moreso, the Greenspan-Guidotti's rule concentrates only on the external drain, without regard to internal influences (Obstfeld et al., 2010). The available alternatives of using the ratio of external reserves to GDP or to broad money has been criticized, due to the stock-flow inconsistency and the impact on money supply or non-uniformity of computing M_2 across countries, respectively. This is especially when the data set is on quarterly basis. The choice of inflation rate as against the

deviation of inflation from a target is due to the fact that Nigeria is yet to adopt inflation targeting framework.

Nigeria being an emerging market economy, like other emerging economies, will definitely have the fear of floating, particularly when exchange rate depreciates, as well as fear of loss in term of decline in reserves, therefore, there is need for exchange rate and foreign interest rate in the extended monetary policy reaction equation, thus the equation becomes:

$$IR_t = \beta_0 + \beta_1 \log R_t + \beta_2 \log IF_t + \beta_3 \log EXR_t + \beta_4 \log FIR_t + \beta_5 Y_t + \mu_t \qquad (3)$$

Where *EXR* is the exchange rate and *FIR* is foreign interest rate, proxied by 3 months treasury bills rate of the US. The choice of exchange rate is determined by the availability of data. Real effective exchange rate is preferred, but the available data does not cover the sample period. Although, this type of linear reaction function is not without its short-comings, but for the purpose of simplicity, it has been widely adopted to explain the actual monetary policy reaction and it does so fairly well.

Furthermore, Genberg and He (2009) argued that a monetary policy reaction to variables other than output and inflation does not necessarily mean that monetary policy targets the variables. The inclusion of the variables in the model simply implies that they transmit information about potential inflation in the economy. Currency depreciation, for instance, can prompt a contractionary monetary policy, because, of its possible impact on domestic prices and not necessarily, because, of an attempt to achieve a target, say a particular level of exchange rate.

Prakash and Willi (2011) viewed the response of monetary policy to external reserves as not an inflationary concern, but mostly due to external and financial fragility effect. The inclusion of foreign interest rate (FIR) is to provide us with information on how the activities in the securities market of Nigeria's major trading partner shape the structure of the country's policy decision making.

III.   Model Specification, Data Issues and Estimation

The Autoregressive Distributed Lag (ARDL) model developed by Pesaran et al (2001) is deployed to estimate equation (3). The choice of ARDL is based on three considerations. First, the model yields a consistent estimate of the long run normal coefficients irrespective of whether the underlying regressors are stationary at I(1) or I(0) or a mixture of the two. Second, it provides unbiased estimates of the long run model, as well as valid t-statistics even when some of the regressors are

endogenous (Harris & Sollis, 2003). Thirdly, it yields high quality results even if the sample size is small.

Following Pesaran et al (2001), therefore, the ARDL format of equation (3) becomes:

$$\Delta logIR_t = \beta_0 + \sum_{i=1}^{m} \beta_1 \Delta logIR_{t-i} + \sum_{i=0}^{n} \beta_2 \Delta logR_{t-i} + \sum_{i=1}^{o} \beta_3 \Delta logIF_{t-i}$$
$$+ \sum_{i=1}^{p} \beta_4 \Delta logEXR_{t-i} + \sum_{i=1}^{q} \beta_5 \Delta logFIR_{t-i} + \sum_{i=1}^{r} \beta_6 \Delta logY_{t-i} + \gamma_1 logIR_{t-1}$$
$$+ \gamma_2 logR_{t-1} + \gamma_3 logIF_{t-1}$$
$$+ \gamma_4 logEXR_{t-1} + \gamma_5 logFIR_{t-1} + \gamma_6 logY_{t-1} + \mu_t \qquad (4)$$

Where t is time, log is the natural logarithm, Δ is the first difference, β_0 is constant, β_1 to β_6 and γ_1 to γ_6 are the coefficients of their respective variables. Other variables are as defined earlier.

According to the Granger representation theorem, when variables are co-integrated, there must also be an error correction model (ECM) that describes the short-run dynamics or adjustment of the co-integrated variables towards their equilibrium values. Hence, the general error correction representation of equation (4) is represented as:

$$\Delta logIR_t = \beta_0 + \sum_{i=1}^{m} \beta_1 \Delta logIR_{t-i} + \sum_{i=0}^{n} \beta_2 \Delta logR_{t-i} + \sum_{i=1}^{o} \beta_3 \Delta logIF_{t-i}$$
$$+ \sum_{i=1}^{p} \beta_4 \Delta logEXR_{t-i} + \sum_{i=1}^{q} \beta_5 \Delta logFIR_{t-i} + \sum_{i=1}^{r} \beta_6 \Delta logY_{t-i}$$
$$+ \gamma EC_{t-1} \qquad (5)$$

Where EC = error correction term from equation (4).

According to Pesaran, et al (2001), two stages are involved in estimating equation (4). First, the null hypothesis of the non-existence of the long run relationship among the variables is defined by H_o: $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = \gamma_5 = \gamma_6 = 0$. H_o is tested against the alternative of H_1: not H_o. rejecting the null hypothesis implies that there exists a long run relationship among the variables irrespective of the integration properties of the variables. This is done by conducting a Wald test with an asymptotic non-standard distribution. Two sets of critical values are tabulated with one set assuming all variables are I(1) and the other I(0). This provides a band covering all possible classifications of the variables into I(1) and I(0). If the calculated F-statistics lies above the upper level of the band, the null hypothesis is rejected, implying that there is co-integration, if it lies below the lower level of the

band; the null cannot be rejected, indicating lack of co-integration. If the F-statistics falls within the band, the result is inconclusive.

III.1   Data Issues and sample Period

To estimate the equation, quarterly data spanning from the period 1999Q1 to 2012Q1 is employed. The data set on interest rate, international reserves (US$Million), inflation and exchange rate are obtained from the publications of the Central Bank of Nigeria (CBN) and National Bureau of Statistics (NBS). The foreign interest rate which is proxied by Treasury bills rate of the US denoted as FIR is sourced from the international financial statistics (IFS) and the output gap denoted by Y is computed using an ideal band-pass filter of Christiano and Fitzgerald (2003). The derivation of the filter follows the analysis in Hens and Kai (2011) as shown in the appendix.

IV.   Empirical Result

Summary statistics and the degree of correlation among the variables in the model were estimated as a preliminary check. Table 1 shows that while the logs of inflation (IF), exchange rate (EXR) and output gap (Y) are positively correlated with interest rate; reserves (R) and foreign interest rate (FIR) are negatively correlated with interest rate. The table also presents the summary statistics of the data used in the estimation equation. There are 51 observations; with the mean of the data spanning from 0.3090 for output gap (Y) to 9.9825 for log of reserves (R).The median is between 0.6627 for FIR and 10.3840 for IR.

Table 1: Summary Statistics of the Variables in the Model

	LOGIR	LOGIF	LOGR	LOGEXR	LOGTBR	Y
Mean	9.9825	2.3562	2.9349	4.8419	0.4211	0.3090
Median	10.3840	2.4989	2.9085	4.8541	0.6627	1.9049
Maximum	11.0362	3.1913	3.2658	5.0643	1.8323	22.2312
Minimum	8.4706	-1.4979	2.6926	4.4998	-2.6736	-29.4067
Std. Dev.	0.8175	0.7153	0.1475	0.1389	1.2711	11.2512
Skewness	-0.5188	-3.3106	0.4700	-0.4934	-0.8541	-0.3750
Kurtosis	1.6730	17.9060	2.5160	2.7893	2.5876	3.3230
Jarque-Bera	6.0291	565.3129	2.3752	2.1639	6.5625	1.4167
Prob.	0.0491	0.0000	0.3049	0.3389	0.0376	0.4924
Obs.	51	51	51	51	51	51

Degree of Corelation among the Variables in the Estimated Equation

	LOGIR	LOGIF	LOGR	LOGEXR	LOGTBR	Y
LOGIR	1.0000	0.1641	-0.8321	0.6781	-0.3406	0.0446
LOGIF		1.0000	0.0247	0.3258	-0.2525	-0.1261
LOGR			1.0000	-0.4563	0.3017	-0.1901
LOGEXR				1.0000	-0.8050	0.1505
LOGFIR					1.0000	-0.1636
Y						1.0000

Although, the ARDL can be applied irrespective of whether the regressors are I(0) or I(1) or a mixture of the two and hence, does not require pretesting of the data, but we decided to determine the order of integration of all the variables before implementing the model, in order to be sure that the data does not contain I(2) series. Table 2 shows the results of the Augmented Dickey-Fuller (based on AIC, SBC and HQ) and Phillips Perron (PP) unit root tests for the order of integration of the variables in the model.

Table 2: Unit-Root Test (Augmented Dickey-Fuller and Phillips-Perron)

Variable	Augmented Dickey Fuller						P-P test statistics	
	AIC		SBC		HQ			
	Level	First Diff.	Level	First Diff.	Level	First Diff.	Level	First Diff.
LIR	-0.897705	-4.923504*	-0.897705	-4.923504*	-0.897705	-4.923504*	-0.478748	-4.910412*
LR	-2.894033	-7.285177*	-2.609695	-7.285177*	-2.609695	-7.285177*	-2.840586	-7.2785*
LIF	-1.051972	-5.320129*	0.669163	-5.320129*	-1.051972	-5.320129*	-0.455639	-3.483824**
LEXR	-2.203007	-5.315952*	-2.203007	-5.315952*	-2.203007	-5.315952*	-2.298330	-5.315952*
LFIR	-2.397748	-2.222297	-2.397748	-2.222297	-2.397748	-2.222297	-1.180669	-2.341412***
Y	-0.816757	-1.862535	-0.816757	-1.862535	-0.816757	-1.862535	-3.259668**	-2.506724

*Notes: * , ** and *** significant at 1%, 5% and 10%, respectively.*

The result of the unit root test shows that, while interest rate (IR), reserves (R), inflation (IF) and exchange rate (EXR) based on the two tests are integrated of I(1) and significant at 1.0 per cent, foreign interest rate (FIR) is of order I(1) based only on the Phillips Perron test but significant at 10.0 per cent. Output gap (Y) is of order I(0) also based only on the Phillips Perron test at 10.0 per cent. Hence, the order of integration of the variables provides further evidence in support of the use of the ARDL model.

Equation (4) is then estimated. Table 3 presents the calculated F-statistics (F-statistics = 67.458) indicating that the null of no co-integration can be rejected at 1.0 per cent level, since it is higher than the upper bound critical value of 4.43 as tabulated   in Pesaran et al (2001). In other words, there exists a long-run relationship or co-integration among the studied variables.

Table 3: Estimated Long-Run Coefficients ARDL (1, 0, 1, 1, 1, 1)

Dependent Variable: **logIR**			
Variables	**Coefficient**	**t-Statistic**	**Prob-Values**
C	0.67957	0.60835	0.54920
LOGIR(-1)	0.82323	15.55993	0.00000
R	-0.03399	-2.74274	0.01190
LOGIF	0.04426	1.59724	0.12450
LOGEXR	0.04759	1.94799	0.06430
LOGFIR	0.40589	1.25298	0.22340
Y	-0.04201	-2.30902	0.03070
R^2 = 0.99	F-Stat = (5, 35) =67.458 [0.0000]		*Durbin Watson* = 2.34
Adjusted - R^2 = 0.98	AIC = -2.110307, SBC = -1.780270 , HQ = -2.006943		

The relevant critical values for unrestricted intercept and no trend under 6 variables for 0.05, 0.025 and 0.01 are 2.45 - 3.61; 2.75 - 3.99 and 3.15 - 4.43, respectively. They are Obtained from Pesaran et al. (2001) CI(iii) Case III.

From Table 3, the result indicates that the long run overall model is well fitted as the independent variables exerts about 98.0 per cent (adjusted-R^2) influence on the dependent variable. The coefficients show that, reserves and output gap exhibit a significant negative relationship with the policy rate (IR). On the other hand, the log of the foreign interest rate (FIR) exhibits a significant positive relationship with the policy rate. While the signs of inflation and exchange rates are positive, although both are not significant.

The negative and statistically significant response[3]of the log of output gap (Y) is consistent with theory and implies an inverse relationship between output gap and policy rate in Nigeria, such that the more actual output in the last quarter

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[3]*This is akin to the finding of Inoue and Hamori on India (2007) and Iklaga on Nigeria (2008)*

falls short of its potential, within the same period (i.e. higher negative output gap), central bank reduces the policy rate, with the notion that the economy has the capacity to absorb more liquidity and vice versa. Although, this is reactionary rather than proactive, but seems to have some degree of semblance with the expected result in a Taylor-type rule with regard to output gap. This also shows a fairly strong relationship between output gap and policy rate in Nigeria. Arguably, therefore, if a large chunk of the real sector activities can be affected by the adjustment in policy rate, this singular reaction of the CBN vis-à-vis policy rate will positively impact on output growth. In practical terms, if the output gap is negative, implying that there is a potential for increasing output or put differently the economy is performing below capacity, CBN can reduce the policy rate, to encourage borrowing, hence boost investment and output. The reverse also follows and this can be interpreted as 'demand effect'.

The significant negative response of policy rate to foreign reserves signifies that, central bank reduces policy rate in response to rise in foreign reserves. Conversely, any fall in foreign reserves coincides with the increase in policy rate. This may not necessarily be in order to attract capital inflows or stop sudden outflows, but also to discourage credit creation by the deposit money banks (DMBs). The coefficient of reserves can be interpreted to mean 'liquidity effect[4]. This is because, by implication as reserves is depleted, especially through intervention in foreign exchange market, more naira is withdrawn from circulation, hence an increase in the policy rate as a response to increase in demand for money or shortage of naira cash balances in the economy.

Although as earlier stated, in the model, inflation is not the main focus as in the original Taylor rule. However, it is important to passively observe that the 'Fisher effect'[5] does not strongly hold for inflation in Nigeria. Even though, the positive sign of the log of inflation would have connoted same, but it is statistically not significant. This, unlike Agu (2007), proves that CBN did not directly react to inflation rate within the study period[6]. This stem probably from two facts: first, that monetary policy in Nigeria, of recent, is conducted on the basis of opinion polls. Second, within the study period, as clearly confirmed by Iklaga (2008), when designing monetary policy, major developments in the economy are reviewed and critical areas that require intervention to enhance output or stabilise prices are identified. Policy interventions are then concentrated on the recognised

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[4] *Liquidity effect asserts that in the short run, changes in the money supply induces changes in short term nominal interest rate in the opposite direction*
[5] *Fisher effect is a theory that postulates a positive relationship between interest rate and inflation*
[6] *Although the sign of the log of inflation is the same as Agu (2007) but the fact that it is not statistically significant informs the difference in the interpretation of the result.*

areas. This focused the conduct of monetary policy mostly to the management of exchange rate, capital inflow, excess liquidity and credit creation.

The high coefficient of the log of policy rate is an indication of interest rate smoothing characteristics of the CBN. Sack and Wieland (2000) are of the opinion that in such a situation, any slight policy adjustment by the central bank would be in near distant future, followed by a relatively larger policy adjustment in the same direction. They attributed interest rate smoothing behaviour of monetary authorities, particularly in the emerging economies, to issues that have to do with stability of the system, adverse reaction of the banking system, as well as effort to protect the reputation of the central bank itself.

The log of exchange rate is positive and fairly statistically significant, implying that some degree of importance is attached to exchange rate in the monetary policy formulation process. The reason could be that since monetary policy is to a large extent anchored on monetary targeting framework, with price stability as the overriding objective, CBN tries to stabilise the exchange rate, so as to avoid speculation in the foreign exchange market, arising from wide premia, which is capable of destabilising the market. The instability or volatility in exchange rate could be counter-productive to the goals of price stability.

The foreign interest rate is positively related to the dependent variable but not statistically significant. This is an indication that, although, central bank monitors development in other economies, in this case, the United States, but hardly changes her policy rate in response to the changes in the foreign interest rate. This probably can be attributed to the effort of the CBN to discourage significant capital outflows or prevent sudden stop in capital inflows.

Table 4: Short-Run Parsimonious Error Correction Estimates of the ARDL Model

Dependent variable: **ΔlogIR**

Regressors	Coefficient	t-Statistic	Prob-Values
C	-0.0075	-0.4176	0.6788
ΔLOGIR(-1)	0.6322	3.6024	0.0010
ΔR	-0.0188	-1.5968	0.1193
ΔR(-1)	-0.0447	-3.4606	0.0014
ΔR(-2)	-0.0172	-1.3942	0.1720
ΔLOGIF(-2)	-0.0413	-1.6160	0.1151
ΔLOGEXR(-1)	0.6249	1.2453	0.2213
ΔLOGEXR(-2)	0.6175	1.3160	0.1967
ΔLOGFIR	0.0601	0.8045	0.4265
ΔY(-1)	0.0048	1.5896	0.1209
ΔY(-2)	-0.0078	-2.4139	0.0211
ECM(-1)	-0.7098	-2.9926	0.0050

R^2 = 0.56	Durbin Watson = 2.01647
Adjusted - R^2 = 0.42	Prob. = 0.0007

The appropriate lag length ρ for the error correction model is selected based on Akaike Information Criterion (AIC), Schwarz Bayesian Criterion (SBC) and Hannan-Quinn Criterion (HQ), as all the tests yielded the same optimal lag length (Table 5). The result of the error correction model (ECM) is presented in Table 4. The coefficient of the error correction term (ECM_{t-1}) is negative and highly significant. This indicates that nearly 30.0 per cent disequilibrium is corrected on quarterly basis by changes in policy rate.

Table 5: Statistics for Selecting Lag Order of the Model

ρ	1	2*	3	4	5
AIC	-1.51408	-1.89094	-1.64317	-1.96383	-3.53269
SBC	-1.05077	-1.18237	-0.67962	-0.73509	-2.04326
HQC	-1.33830	-1.62430	-1.28397	-1.51071	-2.98675

*Note: ρ is the lag order of the model. AIC denotes Akaike Information Criterion, SBC is Schwarz Bayesian Criterion and HQC Hannan Quinn Criterion. * Optimal lag length*

Overall, the result suggests that the simple 'Taylor-type rule' laying emphasis mainly on the output and inflation gaps, although theoretically cogent, is inadequate to track the reactionary tendencies of monetary policy of the CBN.

Thus, the extended version, which considers some other variables, besides inflation and output, is highly necessary to capture the changing pattern of monetary policy decision making process of the CBN, which can largely be attributed to, perhaps the crisis in the banking system during the studied period, as well as the rapidly increasing influence of external factors on the domestic economy.

Meanwhile, a recursive estimate of the level relationship is estimated to test for the tendency of the relationship between the estimated variables to change over time probably due to the variations in the pattern of monetary policy, arising from the emergence of new sources of shocks coupled with changes in the structure of the economy which prompts the monetary authority to often review the *modus operandi* of monetary policy. Although, parameter instability, can in some cases, be attributed to variable omission, but visual inspection of the recursive estimate in the Appendix shows, for instance, that the estimated monetary policy negatively reacts to external reserves (C3) at the beginning of the sample period, but turns positive in 2003, thereafter, became almost flat throughout the remaining part of the sampled period. Similar changes in the pattern of monetary policy reaction, although in different directions, were observed in all the variables throughout the sample period.This type of variations in the parameters in the recursive estimation can in some cases be interpreted to mean policy inconsistency.

The most often used techniques of cumulative sum (CUSUM) and CUSUMSQ tests were adopted to test the stability of the equation and of the estimated parameters.

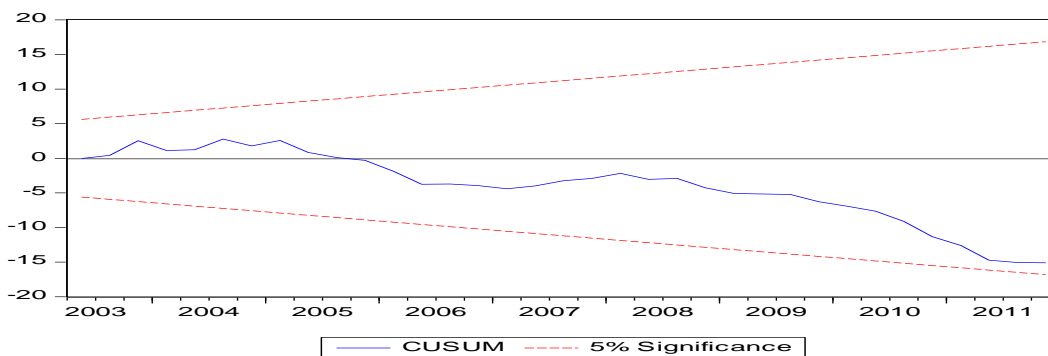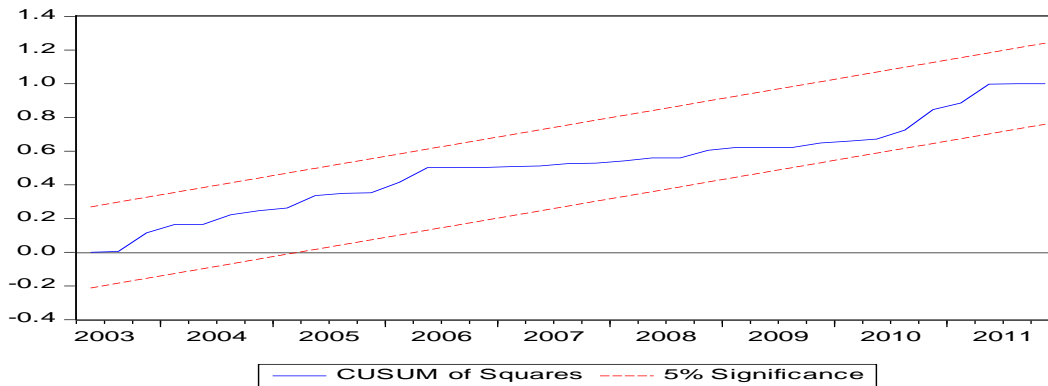**Figure 1: Cumulative Sum (CUSUM) of Recursive Residual Test**

Figure 2: Cumulative Sum of Squares (CUSUMSQ) of Recursive Residual Test

CUSUM test is based on the cumulative sum of the equation errors in the regression. The software represents graphically the cumulative sum of errors together with the critical lines of 5.0 per cent. On the other hand, CUSUMSQ instead used recursive double errors. The equation parameters are considered unstable if the whole sum of recursive errors gets outside the two critical lines. By and large, graphs of CUSUM and CUSUMSQ show that the parameters of the analysed equation are stable given that the recursive errors lie within the two critical lines of both the CUSUM and CUSUMSQ tests.

V. Conclusion

The study applied an Autoregressive Distributed Lag (ARDL) approach to an extended version of the Taylor-type rule to estimate the response of monetary policy in Nigeria to both domestic and external variables, with special emphasis on external reserves. The results show that the Central Bank of Nigeria reacts to changes in the level of external reserves and exchange rate, in addition to output gap, thereby rendered the cogent conventional Taylor rule inadequate to assess the monetary policy reaction function of the Central Bank of Nigeria. This justifies the modification of the rule to incorporate other variables in addition to inflation and output to capture the reaction of monetary policy to developments in the economy. The study validates the interest rate smoothing behavior, showing that the CBN is concerned with costs associated with interest rate variability.

The reaction of the CBN to external reserves is attributed largely to the rapidly increasing influence of external factors on the domestic economy which lures the monetary authority into considering other external sources of vulnerability in the system on the monetary policy decision making process.

It is important to note that monetary policy reaction to variables other than output and inflation does not necessarily imply that monetary policy targets the variables. The inclusion of the variables in the model simply implies that they transmit information about potential output/inflation in the economy.

Finally, there is therefore, the need for the CBN to consider external variables, particularly external reserves and exchange rate when formally designing/building a rule/model for the country's monetary policy.

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Appendix 1. Recursive Estimates of the Monetary Policy Reaction Function
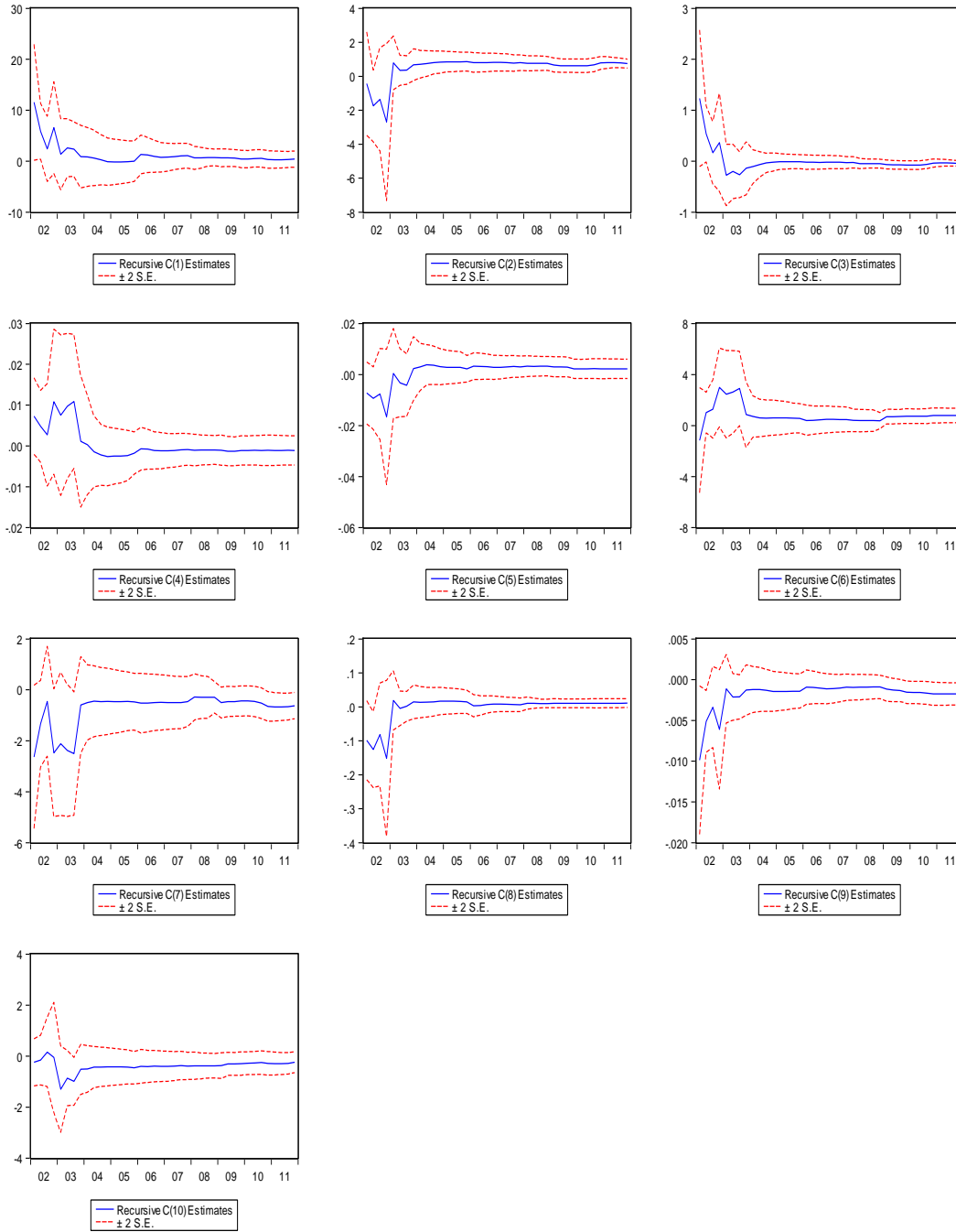
Appendix

Consider a linear filter G(L) which is a linear transformation of a time series x_t with weights g_l at lag l.

$$G(L) = \sum_{l=a}^{b} g_l L^l, \qquad a \le 0 \le b, \qquad (6)$$

Where L is the lag operator $L^k = x_{t-k}$. To produce the filtered series x_t, the filter is applied to y_t:

$$x_t = G(L)y_t = \sum_{l=a}^{b} g_l y_t \qquad (7)$$

The effect of the application of the filter is reflected in the frequency response function (FRF) of the filter. This is represented as:

$$G(e^{-i\omega}) = \sum_{l=a}^{b} g_l e^{i\omega l} \qquad (8)$$

The growth of the amplitudes of y_t is caused by the linear filter.

$$Gain(\omega) = |G(e^{-i\omega})| \qquad (9)$$

While the shift of its position with regards to the phase shift is:

$$Phase(\omega) = \frac{arg\left(G(e^{-i\omega})\right)}{(2\pi)} \quad at \; frequency \; \omega \quad (10)$$

Equations (9) and (10) are respectively the gain shift and phase shift of the filter.

If $g_l = g - l$ for $l > 0$, implying that weights are symmetrical, the linear filter will not cause any phase shift.

However, since phase shift causing filters can lead to either wrong or spurious lead-lag relationships between/among variables, according to Hens and Kai (2011), it therefore, follows that the gain function of the ideal band pass filter is a perfect rectangular function, given as:

$$\text{Gain}(\omega) = \begin{cases} 1 & \text{for } \omega_1 \leq \omega \leq \omega_2 \\ 0 & \text{for } \omega < \omega_1 \text{ or } \omega > \omega_2 \end{cases} \qquad (11)$$

Note that the phase shift function is a constant zero.

Now, to derive the weight of the ideal band pass filter, we have:

$$g_l = \begin{cases} \dfrac{\sin(\omega_2 l) - \sin(\omega_1 l)}{\pi l} & \text{for } l \neq 0 \\ \dfrac{\omega_2 - \omega_1}{\pi} & \text{for } 1 \neq 0 \end{cases} \qquad (12)$$

However, according to Hens and Kai (2011), the ideal band pass filter is practically not feasible, because, of the infinite nature of the weights, In other words, to calculate such a filter an infinite-order moving average is necessary which requires a data set of infinite length and this is practically not available. Therefore, some form of approximation is required, thus making the contributions of Baxter and King (1999) and Christiano and Fitzgerald (2003) highly relevant[7].

Christiano-Fitzgerald[8] approximation, uses alternative loss criterion, as well as, the assumptions on the underlying process of y_t they yielded, to adjust the weights to take account of the missing values. The extrapolation of the sample is done by using what is referred to as 'an assumed model' and the extrapolation overlaps the observed sample. Now, following Hens and Kai (2011), if we assumed a random walk for series y_t, the following simple adjustment is required:

$$\check{g}_0 = \frac{g_0}{2} \qquad (13)$$

$$\check{g}_1 = \frac{-g_0}{2} \qquad (14)$$

$$\check{g}_1 = \frac{-g_0}{2} - \sum_{k=1}^{l-1} g_l, l \geq 2 \qquad (15)$$

Where g_l in equation (15) is the weight of the ideal filter, as represented in equation (12). The $\check{g}_1$ is the adjusted weights and are used on the end points y_1 and y_T. In between are the observations that are weighted by the unmodified weights g_l.

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[7]*Emphasis in on Christiano-Fitzgerald approximation.*
[8] *For detailed exploration on the derivation of BK and CF filter, see Hens and Kai (2011) and Christiano and Fitzgerald (2003)*